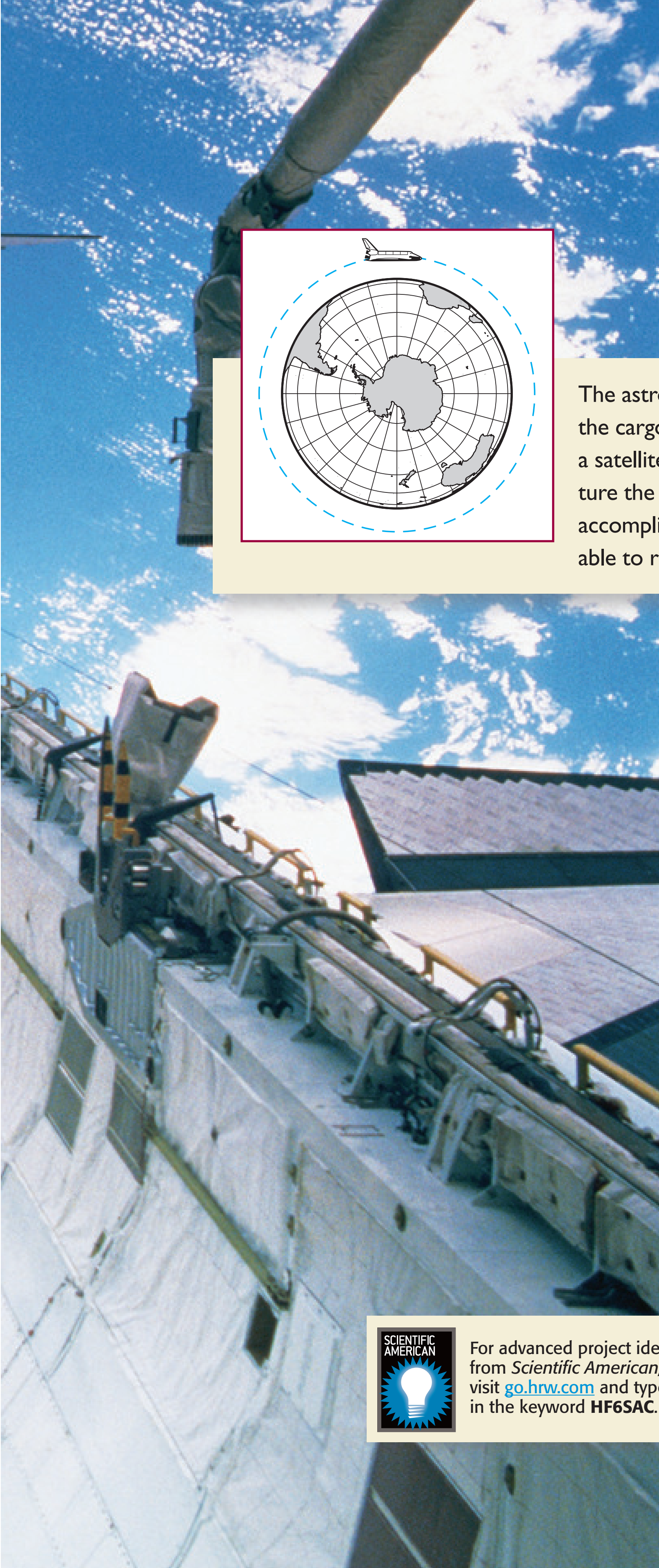


CHAPTER 7

Circular Motion and Gravitation

The astronaut shown in this photograph is walking out onto the cargo bay area of the space shuttle to attempt repair of a satellite. Although the astronaut's initial attempt to capture the satellite was unsuccessful, this task was later accomplished by a robotic arm. The astronauts were then able to repair the satellite.

WHAT TO EXPECT

In this chapter, you will learn how to describe circular motion and the forces associated with it, including the force due to gravity.

WHY IT MATTERS

Circular motion is present all around you—from a rotating Ferris wheel in an amusement park to a space shuttle orbiting Earth to Earth's orbit around the sun.

CHAPTER PREVIEW

SCIENTIFIC AMERICAN

For advanced project ideas from *Scientific American*, visit go.hrw.com and type in the keyword **HF6SAC**.

SECTION 1

Circular Motion

SECTION OBJECTIVES

- **Solve problems involving centripetal acceleration.**
- **Solve problems involving centripetal force.**
- **Explain how the apparent existence of an outward force in circular motion can be explained as inertia resisting the centripetal force.**

CENTRIPETAL ACCELERATION

Consider a spinning Ferris wheel, as shown in **Figure 1.** The cars on the rotating Ferris wheel are said to be in *circular motion.* Any object that revolves about a single axis undergoes circular motion. The line about which the rotation occurs is called the *axis of rotation.* In this case, it is a line perpendicular to the side of the Ferris wheel and passing through the wheel's center.

Tangential speed depends on distance

Tangential speed (v_t) can be used to describe the speed of an object in circular motion. The tangential speed of a car on the Ferris wheel is the car's speed along an imaginary line drawn tangent to the car's circular path. This definition can be applied to any object moving in circular motion. When the tangential speed is constant, the motion is described as *uniform circular motion.*

The tangential speed depends on the distance from the object to the center of the circular path. For example, consider a pair of horses side-by-side on a carousel. Each completes one full circle in the same time period, but the horse on the outside covers more distance than the inside horse does, so the outside horse has a greater tangential speed.

Centripetal acceleration is due to a change in direction

Suppose a car on a Ferris wheel is moving at a constant speed around the wheel. Even though the tangential speed is constant, the car still has an acceleration. To see why, consider the equation that defines acceleration:

$$\mathbf{a} = \frac{\mathbf{v_f} - \mathbf{v_i}}{t_f - t_i}$$

Acceleration depends on a change in the velocity. Because velocity is a vector, acceleration can be produced by a change in the *magnitude* of the velocity, a change in the *direction* of the velocity, or both.

Figure 1
Any point on a Ferris wheel spinning about a fixed axis undergoes circular motion.

ADVANCED TOPICS

See "Tangential Speed and Acceleration" in **Appendix J: Advanced Topics** to learn more about tangential speed, and to be introduced to the concept of tangential acceleration.

The acceleration of a Ferris wheel car moving in a circular path and at constant speed is due to a change in direction. An acceleration of this nature is called a **centripetal acceleration.** The magnitude of a centripetal acceleration is given by the following equation:

centripetal acceleration

the acceleration directed toward the center of a circular path

CENTRIPETAL ACCELERATION

$$a_c = \frac{v_t^2}{r}$$

$$\text{centripetal acceleration} = \frac{(\text{tangential speed})^2}{\text{radius of circular path}}$$

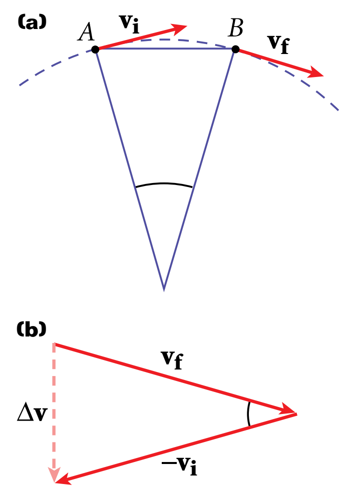


Figure 2
(a) As the particle moves from *A* to *B*, the direction of the particle's velocity vector changes. **(b)** For short time intervals, $\Delta\mathbf{v}$ is directed toward the center of the circle.

What is the direction of centripetal acceleration? To answer this question, consider **Figure 2(a).** At time t_i, an object is at point A and has tangential velocity $\mathbf{v_i}$. At time t_f, the object is at point B and has tangential velocity $\mathbf{v_f}$. Assume that $\mathbf{v_i}$ and $\mathbf{v_f}$ differ in direction but have the same magnitudes.

The change in velocity ($\Delta\mathbf{v} = \mathbf{v_f} - \mathbf{v_i}$) can be determined graphically, as shown by the vector triangle in **Figure 2(b).** Note that when Δt is very small, $\mathbf{v_f}$ will be almost parallel to $\mathbf{v_i}$. The vector $\Delta\mathbf{v}$ will be approximately perpendicular to $\mathbf{v_f}$ and $\mathbf{v_i}$ and will be pointing toward the center of the circle. Because the acceleration is in the direction of $\Delta\mathbf{v}$, the acceleration will also be directed toward the center of the circle. Centripetal acceleration is always directed toward the center of a circle. In fact, the word *centripetal* means "center seeking." This is the reason that the acceleration of an object in uniform circular motion is called *centripetal acceleration.*

SAMPLE PROBLEM A

Centripetal Acceleration

PROBLEM

A test car moves at a constant speed around a circular track. If the car is 48.2 m from the track's center and has a centripetal acceleration of 8.05 m/s^2, what is the car's tangential speed?

SOLUTION

Given: $r = 48.2$ m $\quad a_c = 8.05$ m/s^2

Unknown: $v_t = ?$

Use the centripetal acceleration equation, and rearrange to solve for v_t.

$$a_c = \frac{v_t^2}{r}$$

$$v_t = \sqrt{a_c r} = \sqrt{(8.05 \text{ m/s}^2)(48.2 \text{ m})}$$

$$\boxed{v_t = 19.7 \text{ m/s}}$$

PRACTICE A

Centripetal Acceleration

1. A rope attaches a tire to an overhanging tree limb. A girl swinging on the tire has a centripetal acceleration of 3.0 m/s^2. If the length of the rope is 2.1 m, what is the girl's tangential speed?

2. As a young boy swings a yo-yo parallel to the ground and above his head, the yo-yo has a centripetal acceleration of 250 m/s^2. If the yo-yo's string is 0.50 m long, what is the yo-yo's tangential speed?

3. A dog sits 1.5 m from the center of a merry-go-round. The merry-go-round is set in motion, and the dog's tangential speed is 1.5 m/s. What is the dog's centripetal acceleration?

4. A race car moving along a circular track has a centripetal acceleration of 15.4 m/s^2. If the car has a tangential speed of 30.0 m/s, what is the distance between the car and the center of the track?

Tangential acceleration is due to a change in speed

You have seen that centripetal acceleration results from a change in direction. In circular motion, an acceleration due to a change in speed is called *tangential acceleration.* To understand the difference between centripetal and tangential acceleration, consider a car traveling in a circular track. Because the car is moving in a circle, the car has a centripetal component of acceleration. If the car's speed changes, the car also has a tangential component of acceleration.

Figure 3
When a ball is whirled in a circle, it is acted on by a force directed toward the center of the ball's circular path.

CENTRIPETAL FORCE

Consider a ball of mass m that is tied to a string of length r and that is being whirled in a horizontal circular path, as shown in **Figure 3.** Assume that the ball moves with constant speed. Because the velocity vector, **v,** continuously changes direction during the motion, the ball experiences a centripetal acceleration that is directed toward the center of motion. As seen earlier, the magnitude of this acceleration is given by the following equation:

$$a_c = \frac{v_t^2}{r}$$

The inertia of the ball tends to maintain the ball's motion in a straight path. However, the string exerts a force that overcomes this tendency. The forces acting on the ball are gravitational force and the force exerted by the string, as shown in **Figure 4(a)** on the next page. The force exerted by the string has horizontal and vertical components. The vertical component is equal and opposite to the gravitational force. Thus, the horizontal component is the net force. This net force is directed toward the center of the circle, as shown in **Figure 4(b).** The net force that is directed toward the center of an object's circular path is called *centripetal force.* Newton's second law can be applied to find the magnitude of this force.

$$F_c = ma_c$$

The equation for centripetal acceleration can be combined with Newton's second law to obtain the following equation for centripetal force:

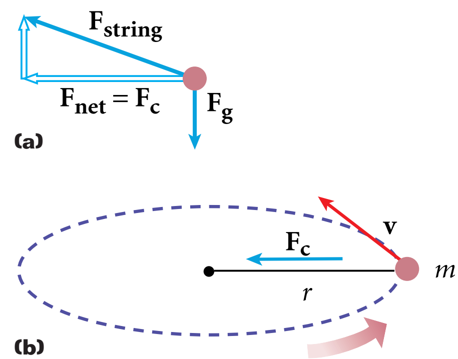


Figure 4
The net force on a ball whirled in a circle **(a)** is directed toward the center of the circle **(b).**

CENTRIPETAL FORCE

$$F_c = \frac{mv_t^2}{r}$$

$$\textbf{centripetal force} = \textbf{mass} \times \frac{\textbf{(tangential speed)}^2}{\textbf{radius of circular path}}$$

Centripetal force is simply the name given to the net force on an object in uniform circular motion. Any type of force or combination of forces can provide this net force. For example, friction between a race car's tires and a circular track is a centripetal force that keeps the car in a circular path. As another example, gravitational force is a centripetal force that keeps the moon in its orbit.

SAMPLE PROBLEM B

Centripetal Force

PROBLEM

A pilot is flying a small plane at 56.6 m/s in a circular path with a radius of 188.5 m. The centripetal force needed to maintain the plane's circular motion is 1.89×10^4 N. What is the plane's mass?

SOLUTION

Given: $v_t = 56.6$ m/s $r = 188.5$ m $F_c = 1.89 \times 10^4$ N

Unknown: $m = ?$

Use the equation for centripetal force. Rearrange to solve for *m*.

$$F_c = \frac{mv_t^2}{r}$$

$$m = \frac{F_c r}{v_t^2} = \frac{(1.89 \times 10^4 \text{ N})(188.5 \text{ m})}{(56.6 \text{ m/s})^2}$$

$$\boxed{m = 1110 \text{ kg}}$$

PRACTICE B

Centripetal Force

1. A 2.10 m rope attaches a tire to an overhanging tree limb. A girl swinging on the tire has a tangential speed of 2.50 m/s. If the magnitude of the centripetal force is 88.0 N, what is the girl's mass?

2. A bicyclist is riding at a tangential speed of 13.2 m/s around a circular track. The magnitude of the centripetal force is 377 N, and the combined mass of the bicycle and rider is 86.5 kg. What is the track's radius?

3. A dog sits 1.50 m from the center of a merry-go-round and revolves at a tangential speed of 1.80 m/s. If the dog's mass is 18.5 kg, what is the magnitude of the centripetal force on the dog?

4. A 905 kg car travels around a circular track with a circumference of 3.25 km. If the magnitude of the centripetal force is 2140 N, what is the car's tangential speed?

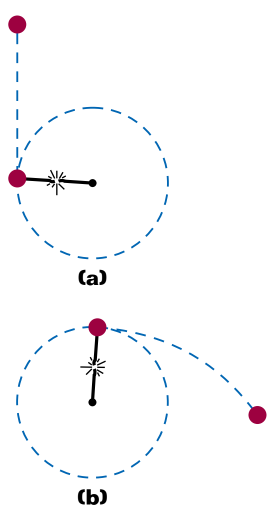


Figure 5
A ball that is on the end of a string is whirled in a vertical circular path. If the string breaks at the position shown in **(a)**, the ball will move vertically upward in free fall. **(b)** If the string breaks at the top of the ball's path, the ball will move along a parabolic path.

Centripetal force is necessary for circular motion

Because centripetal force acts at right angles to an object's circular motion, the force changes the direction of the object's velocity. If this force vanishes, the object stops moving in a circular path. Instead, the object moves along a straight path that is tangent to the circle.

For example, consider a ball that is attached to a string and that is whirled in a vertical circle, as shown in **Figure 5.** If the string breaks when the ball is at the position shown in **Figure 5(a),** the centripetal force will vanish. Thus, the ball will move vertically upward, as if it has been thrown straight up in the air. If the string breaks when the ball is at the top of its circular path, as shown in **Figure 5(b),** the ball will fly off horizontally in a direction tangent to the path. The ball will then move in the parabolic path of a projectile.

DESCRIBING A ROTATING SYSTEM

To better understand the motion of a rotating system, consider a car traveling at high speed and approaching an exit ramp that curves to the left. As the driver makes the sharp left turn, the passenger slides to the right and hits the door. At that point, the force of the door keeps the passenger from being ejected from the car. What causes the passenger to move toward the door? A popular explanation is that a force must push the passenger outward. This force is sometimes called the *centrifugal force,* but that term often creates confusion, so it is not used in this textbook.

Inertia is often misinterpreted as a force

The phenomenon is correctly explained as follows: Before the car enters the ramp, the passenger is moving in a straight path. As the car enters the ramp and travels along a curved path, the passenger, because of inertia, tends to move along the original straight path. This movement is in accordance with Newton's first law, which states that the natural tendency of a body is to continue moving in a straight line.

However, if a sufficiently large centripetal force acts on the passenger, the person will move along the same curved path that the car does. The origin of the centripetal force is the force of friction between the passenger and the car seat. If this frictional force is not sufficient, the passenger slides across the seat as the car turns underneath. Eventually, the passenger encounters the door, which provides a large enough force to enable the passenger to follow the same curved path as the car does. The passenger does not slide toward the door because of some mysterious outward force. Instead, the frictional force exerted on the passenger by the seat is not great enough to keep the passenger moving in the same circle as the car.

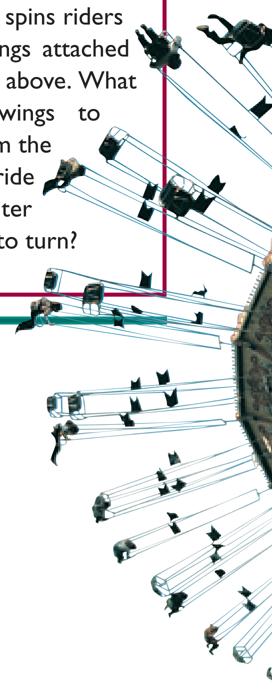
Conceptual Challenge

1. Pizza

Pizza makers traditionally form the crust by throwing the dough up in the air and spinning it. Why does this make the pizza crust bigger?

2. Swings

The amusement-park ride pictured below spins riders around on swings attached by cables from above. What causes the swings to move away from the center of the ride when the center column begins to turn?



SECTION REVIEW

1. What are three examples of circular motion?
2. A girl on a spinning amusement park ride is 12 m from the center of the ride and has a centripetal acceleration of 17 m/s^2. What is the girl's tangential speed?
3. Use an example to describe the difference between tangential and centripetal acceleration.
4. Identify the forces that contribute to the centripetal force on the object in each of the following examples:
   a. a *bicyclist* moving around a flat, circular track
   b. a *bicycle* moving around a flat, circular track
   c. a *race car* turning a corner on a steeply banked curve
5. A 90.0 kg person rides a spinning amusement park ride that has a radius of 11.5 m. If the person's tangential speed is 13.2 m/s, what is the magnitude of the centripetal force acting on the person?
6. Explain what makes a passenger in a turning car slide toward the door of the car.
7. **Critical Thinking** A roller coaster's passengers are suspended upside down as it moves at a constant speed through a vertical loop. What is the direction of the force that causes the coaster and its passengers to move in a circle? What provides this force?

SECTION 2

Newton's Law of Universal Gravitation

SECTION OBJECTIVES

- **Explain how Newton's law of universal gravitation accounts for various phenomena, including satellite and planetary orbits, falling objects, and the tides.**
- **Apply Newton's law of universal gravitation to solve problems.**

GRAVITATIONAL FORCE

Earth and many of the other planets in our solar system travel in nearly circular orbits around the sun. Thus, a centripetal force must keep them in orbit. One of Isaac Newton's great achievements was the realization that the centripetal force that holds the planets in orbit is the very same force that pulls an apple toward the ground—**gravitational force.**

gravitational force

the mutual force of attraction between particles of matter

Orbiting objects are in free fall

To see how this idea is true, we can use a thought experiment that Newton developed. Consider a cannon sitting on a high mountaintop, as shown in **Figure 6.** The path of each cannonball is a parabola, and the horizontal distance that each cannonball covers increases as the cannonball's initial speed increases. Newton realized that if an object were projected at just the right speed, the object would fall down toward Earth in just the same way that Earth curved out from under it. In other words, it would orbit Earth. In this case, the gravitational force between the cannonball and Earth is a centripetal force that keeps the cannonball in orbit. Satellites stay in orbit for this same reason. Thus, the force that pulls an apple toward Earth is the same force that keeps the moon and other satellites in orbit around Earth. Similarly, a gravitational attraction between Earth and our sun keeps Earth in its orbit around the sun.

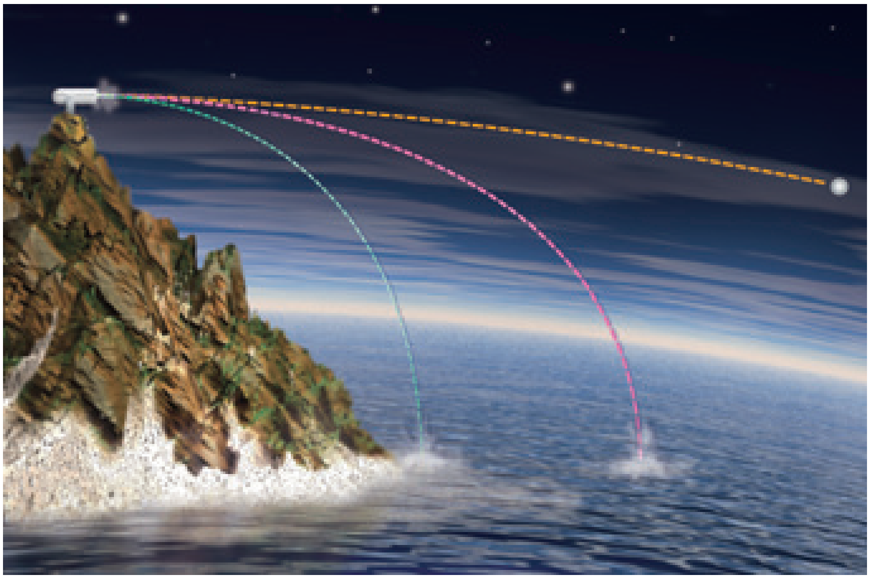


Figure 6
Each successive cannonball has a greater initial speed, so the horizontal distance that the ball travels increases. If the initial speed is great enough, the curvature of Earth will cause the cannonball to continue falling without ever landing.

Gravitational force depends on the masses and the distance

Newton developed the following equation to describe quantitatively the magnitude of the gravitational force if distance r separates masses m_1 and m_2:

NEWTON'S LAW OF UNIVERSAL GRAVITATION

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$\textbf{gravitational force} = \textbf{constant} \times \frac{\textbf{mass 1} \times \textbf{mass 2}}{(\textbf{distance between masses})^2}$$

G is called the *constant of universal gravitation.* The value of G was unknown in Newton's day, but experiments have since determined the value to be as follows:

$$G = 6.673 \times 10^{-11} \frac{\mathrm{N \bullet m^2}}{\mathrm{kg^2}}$$

Newton demonstrated that the gravitational force that a spherical mass exerts on a particle outside the sphere would be the same if the entire mass of the sphere were concentrated at the sphere's center. When calculating the gravitational force between Earth and our sun, for example, you use the distance between their centers.

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Topic: Gravity and Orbiting Objects

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Gravitational force acts between all masses

Gravitational force always attracts objects to one another, as shown in **Figure 7.** The force that the moon exerts on Earth is equal and opposite to the force that Earth exerts on the moon. This relationship is an example of Newton's third law of motion. Also, note that the gravitational forces shown in **Figure 7** are centripetal forces. As a result of these centripetal forces, the moon and Earth each orbit around the center of mass of the Earth-moon system. Because Earth has a much greater mass than the moon, this center of mass lies within Earth.

Gravitational force exists between any two masses, regardless of size. For instance, desks in a classroom have a mutual attraction because of gravitational force. The force between the desks, however, is negligibly small relative to the force between each desk and Earth because of the differences in mass.

If gravitational force acts between all masses, why doesn't Earth accelerate up toward a falling apple? In fact, it does! But, Earth's acceleration is so tiny that you cannot detect it. Because Earth's mass is so large and acceleration is inversely proportional to mass, the Earth's acceleration is negligible. The apple has a much smaller mass and thus a much greater acceleration.

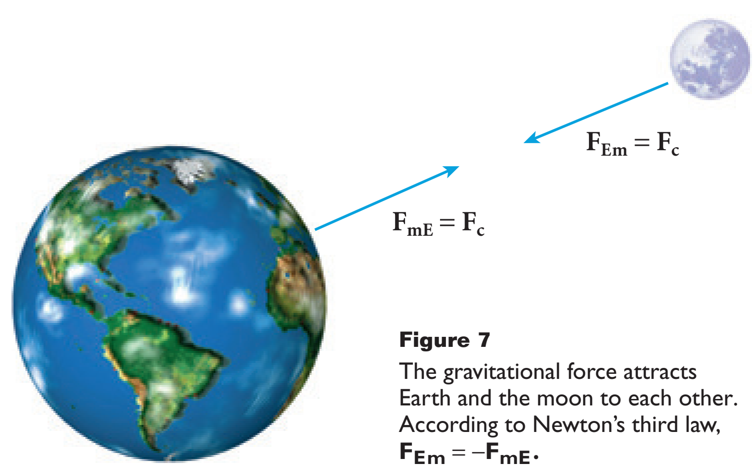


Figure 7
The gravitational force attracts Earth and the moon to each other. According to Newton's third law, $\mathbf{F_{Em}} = -\mathbf{F_{mE}}$.

SAMPLE PROBLEM C

Gravitational Force

PROBLEM

Find the distance between a 0.300 kg billiard ball and a 0.400 kg billiard ball if the magnitude of the gravitational force between them is 8.92×10^{-11} N.

SOLUTION

Given: $m_1 = 0.300$ kg $\quad m_2 = 0.400$ kg $\quad F_g = 8.92 \times 10^{-11}$ N

Unknown: $r = ?$

Use the equation for Newton's law of universal gravitation, and solve for *r*.

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$r = \sqrt{G\frac{m_1 m_2}{F_g}}$$

$$r = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right) \times \frac{(0.300 \text{ kg})(0.400 \text{ kg})}{8.92 \times 10^{-11} \text{ N}}}$$

$$\boxed{r = 3.00 \times 10^{-1} \text{ m}}$$

PRACTICE C

Gravitational Force

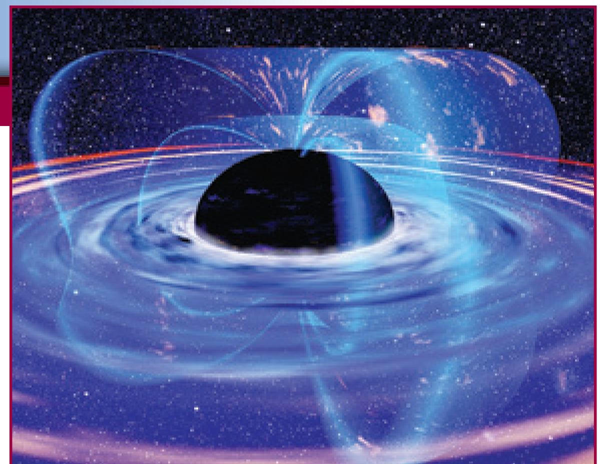
1. What must be the distance between two 0.800 kg balls if the magnitude of the gravitational force between them is equal to that in Sample Problem C?

2. Mars has a mass of about 6.4×10^{23} kg, and its moon Phobos has a mass of about 9.6×10^{15} kg. If the magnitude of the gravitational force between the two bodies is 4.6×10^{15} N, how far apart are Mars and Phobos?

3. Find the magnitude of the gravitational force a 66.5 kg person would experience while standing on the surface of each of the following planets:

Planet	Mass	Radius
a. Earth	5.97×10^{24} kg	6.38×10^{6} m
b. Mars	6.42×10^{23} kg	3.40×10^{6} m
c. Pluto	1.25×10^{22} kg	1.20×10^{6} m

The Inside Story on Black Holes



This artist's conception shows a disk of material orbiting a black hole. Such disks provide indirect evidence of black holes within our own galaxy.

A black hole is an object that is so massive that nothing, not even light, can escape the pull of its gravity. In 1916, Karl Schwarzschild was the first person to suggest the existence of black holes. He used his solutions to Einstein's general-relativity equations to explain the properties of black holes. In 1967, the physicist John Wheeler coined the term "*black hole*" to describe these objects.

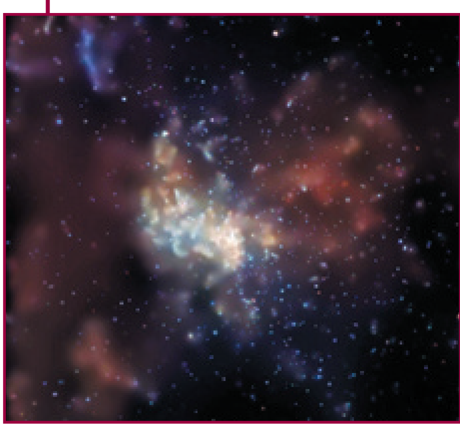
In order for an object to escape the gravitational pull of a planet, such as Earth, the object must be moving away from the planet faster than a certain threshold speed, which is called the *escape velocity.* The escape velocity at the surface of Earth is about 1.1×10^4 m/s, or about 25 000 mi/h.

The escape velocity for a black hole is greater than the speed of light. And, according to Einstein's special theory of relativity, no object can move at a speed equal to or greater than the speed of light. Thus, no object that is within a certain distance of a black hole can move fast enough to escape the gravitational pull of the black hole. That distance, called the *Schwarzschild radius,* defines the edge, or *horizon,* of a black hole.

How can a black hole trap light if light has no mass? According to Einstein's general theory of relativity, any object with mass bends the fabric of space and time itself. When an object that has mass or even when a ray of light passes near another object, the path of the moving object or ray curves because space-time itself is curved. The curvature is so great inside a black hole that the path of any light that might be emitted from the black hole bends back toward the black hole and remains trapped inside the horizon.

Because black holes trap light, they cannot be observed directly. Instead, astronomers must look for indirect evidence of black holes. For example, astronomers have observed stars orbiting very rapidly around the centers of some galaxies. By measuring the speed of the orbits, astronomers can calculate the mass of the dark object—the black hole—that must be at the galaxy's center. Black holes at the centers of galaxies typically have masses millions or billions of times the mass of the sun.

The figure above shows a disk of material orbiting a black hole. Material that orbits a black hole can move at such high speeds and have so much energy that the material emits X rays. From observations of the X rays coming from such disks, scientists have discovered several black holes within our own galaxy.



This image from NASA's *Chandra X-ray Observatory* is of Sagittarius A*, which is a supermassive black hole at the center of our galaxy. Astronomers are studying the image to learn more about Sagittarius A* and about black holes in the centers of other galaxies.

APPLYING THE LAW OF GRAVITATION

For about six hours, water slowly rises along the shoreline of many coastal areas and culminates in a high tide. The water level then slowly lowers for about six hours and returns to a low tide. This cycle then repeats. Tides take place in all bodies of water but are most noticeable along seacoasts. In the Bay of Fundy, shown in **Figure 8,** the water rises as much as 16 m from its low point. Because a high tide happens about every 12 hours, there are usually two high tides and two low tides each day. Before Newton developed the law of universal gravitation, no one could explain why tides occur in this pattern.

Did you know?

When the sun and moon are in line, the combined effect produces a greater-than-usual high tide called a *spring tide.* When the sun and moon are at right angles, the result is a lower-than-normal high tide called a *neap tide.* Each revolution of the moon around Earth corresponds to two spring tides and two neap tides.

Newton's law of gravitation accounts for ocean tides

High and low tides are partly due to the gravitational force exerted on Earth by its moon. The tides result from the *difference* between the gravitational force at Earth's surface and at Earth's center. A full explanation is beyond the scope of this text, but we will briefly examine this relationship.

The two high tides take place at locations on Earth that are nearly in line with the moon. On the side of Earth that is nearest to the moon, the moon's gravitational force is *greater* than it is at Earth's center (because gravitational force decreases with distance). The water is pulled toward the moon, creating an outward bulge. On the opposite side of Earth, the gravitational force is *less* than it is at the center. On this side, all mass is still pulled toward the moon, but the water is pulled least. This creates another outward bulge. Two high tides take place each day because when Earth rotates one full time, any given point on Earth will pass through both bulges.

The moon's gravitational force is not the only factor that affects ocean tides. Other influencing factors include the depths of the ocean basins, Earth's tilt and rotation, and friction between the ocean water and the ocean floor. The sun also contributes to Earth's ocean tides, but the sun's effect is not as significant as the moon's is. Although the sun exerts a much greater gravitational force on Earth than the moon does, the *difference* between the force on the far and near sides of Earth is what affects the tides.

Figure 8
Some of the world's highest tides occur at the Bay of Fundy, which is between New Brunswick and Nova Scotia, Canada. These photographs show a river outlet to the Bay of Fundy at low and high tide.

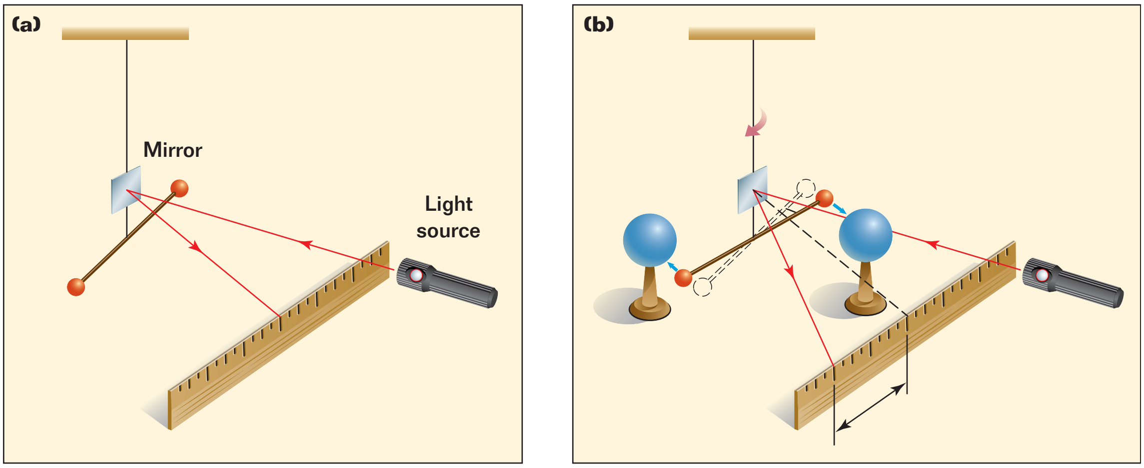


Figure 9
Henry Cavendish used an experiment similar to this one to determine the value of G.

Cavendish finds the value of *G* and Earth's mass

In 1798, Henry Cavendish conducted an experiment that determined the value of the constant G. This experiment is illustrated in **Figure 9.** As shown in **Figure 9(a),** two small spheres are fixed to the ends of a suspended light rod. These two small spheres are attracted to two larger spheres by the gravitational force, as shown in **Figure 9(b).** The angle of rotation is measured with a light beam and is then used to determine the gravitational force between the spheres. When the masses, the distance between them, and the gravitational force are known, Newton's law of universal gravitation can be used to find G. Once the value of G is known, the law can be used again to find Earth's mass.

Gravity is a field force

Newton was not able to explain how objects can exert forces on one another without coming into contact. He developed a mathematical theory to describe gravity, but he did not have a physical explanation for how gravity works. Scientists later developed a theory of fields to explain how gravity and other field forces operate. According to this theory, masses create a gravitational field in space. (Similarly, charged objects generate an electric field.) A gravitational force is an interaction between a mass and the gravitational field created by other masses.

When you raise a ball to a certain height above Earth, the ball gains potential energy. Where is this potential energy stored? The physical properties of the ball and of Earth have not changed. However, the gravitational field between the ball and Earth *has* changed since the ball has changed position relative to Earth. According to field theory, the gravitational energy is stored in the gravitational field itself.

At any point, Earth's gravitational field can be described by the *gravitational field strength,* abbreviated g. The value of g is equal to the magnitude of the gravitational force exerted on a unit mass at that point, or $g = F_g/m$. The gravitational field **(g)** is a vector with a magnitude of g that points in the direction of the gravitational force.

Quick Lab

Gravitational Field Strength

MATERIALS LIST

- spring scale
- hook (of a known mass)
- various masses

You can attach a mass to a spring scale to find the gravitational force that is acting on that mass. Attach various combinations of masses to the hook, and record the force in each case. Use your data to calculate the gravitational field strength for each trial ($g = F_g/m$). Be sure that your calculations account for the mass of the hook. Average your values to find the gravitational field strength at your location on Earth's surface. Do you notice anything about the value you obtained?

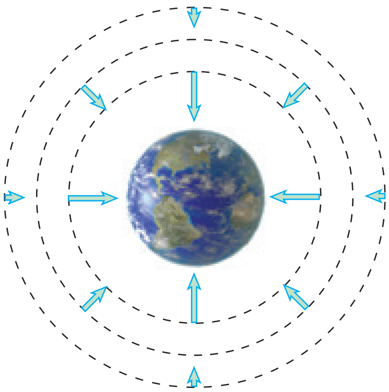


Figure 10
The gravitational field vectors represent Earth's gravitational field at each point. Note that the field has the same strength at equal distances from Earth's center.

Gravitational field strength equals free-fall acceleration

Consider an object that is free to accelerate and is acted on only by gravitational force. According to Newton's second law, $\mathbf{a} = \mathbf{F}/m$. As seen earlier, $\mathbf{g}$ is defined as $\mathbf{F_g}/m$, where $\mathbf{F_g}$ is gravitational force. Thus, the value of g at any given point is equal to the acceleration due to gravity. For this reason, $g = 9.81\ m/s^2$ on Earth's surface. Although gravitational field strength and free-fall acceleration are equivalent, they are not the same thing. For instance, when you hang an object from a spring scale, you are measuring gravitational field strength. Because the mass is at rest (in a frame of reference fixed to Earth's surface), there is no measurable acceleration.

Figure 10 shows gravitational field vectors at different points around Earth. As shown in the figure, gravitational field strength rapidly decreases as the distance from Earth increases, as you would expect from the inverse-square nature of Newton's law of universal gravitation.

Weight changes with location

In the chapter about forces, you learned that weight is the magnitude of the force due to gravity, which equals mass times free-fall acceleration. We can now refine our definition of weight as mass times gravitational field strength. The two definitions are mathematically equivalent, but our new definition helps to explain why your weight changes with your location in the universe.

Newton's law of universal gravitation shows that the value of g depends on mass and distance. For example, consider a tennis ball of mass m. The gravitational force between the tennis ball and Earth is as follows:

$$F_g = \frac{Gmm_E}{r^2}$$

Combining this equation with the definition for gravitational field strength yields the following expression for g:

$$g = \frac{F_g}{m} = \frac{Gmm_E}{mr^2} = G\frac{m_E}{r^2}$$

This equation shows that gravitational field strength depends only on mass and distance. Thus, as your distance from Earth's center increases, the value of g decreases, so your weight also decreases. On the surface of any planet, the value of g, as well as your weight, will depend on the planet's mass and radius.

Conceptual Challenge

1. Gravity on the Moon The magnitude of g on the moon's surface is about $\frac{1}{6}$ of the value of g on Earth's surface. Can you infer from this relationship that the moon's mass is $\frac{1}{6}$ of Earth's mass? Why or why not?

2. Selling Gold A scam artist hopes to make a profit by buying and selling gold at different altitudes for the same price per weight. Should the scam artist buy or sell at the higher altitude? Explain.

Gravitational mass equals inertial mass

Because gravitational field strength equals free-fall acceleration, free-fall acceleration on the surface of Earth likewise depends only on Earth's mass and radius. Free-fall acceleration does not depend on the falling object's mass, because m cancels from each side of the equation, as shown on the previous page.

Although we are assuming that the m in each equation is the same, this assumption was not always an accepted scientific fact. In Newton's second law, m is sometimes called *inertial mass* because this m refers to the property of an object to resist acceleration. In Newton's gravitation equation, m is sometimes called *gravitational mass* because this m relates to how objects attract one another.

How do we know that inertial and gravitational mass are equal? The fact that the acceleration of objects in free fall on Earth's surface is always the same confirms that the two types of masses are equal. A more massive object experiences a greater gravitational force, but the object resists acceleration by just that amount. For this reason, all masses fall with the same acceleration (disregarding air resistance).

There is no obvious reason why the two types of masses should be equal. For instance, the property of electric charges that causes them to be attracted or repelled was originally called *electrical mass*. Even though this term has the word *mass* in it, electrical *mass* has no connection to gravitational or inertial mass. The equality between inertial and gravitational mass has been continually tested and has thus far always held up.

ADVANCED TOPICS

The equality of gravitational and inertial masses puzzled scientists for many years. Einstein's general theory of relativity was the first explanation of this equality. See "General Relativity" in **Appendix J: Advanced Topics** to learn more about this topic.

SECTION REVIEW

1. Explain how the force due to gravity keeps a satellite in orbit.

2. Is there gravitational force between two students sitting in a classroom? If so, explain why you don't observe any effects of this force.

3. Earth has a mass of 5.97×10^{24} kg and a radius of 6.38×10^{6} m, while Saturn has a mass of 5.68×10^{26} kg and a radius of 6.03×10^{7} m. Find the weight of a 65.0 kg person at the following locations:

a. on the surface of Earth
b. 1000 km above the surface of Earth
c. on the surface of Saturn
d. 1000 km above the surface of Saturn

4. What is the magnitude of g at a height above Earth's surface where free-fall acceleration equals 6.5 m/s^2?

5. **Critical Thinking** Suppose the value of G has just been discovered. Use the value of G and an approximate value for Earth's radius (6.38×10^{6} m) to find an approximation for Earth's mass.

SECTION 3

Motion in Space

SECTION OBJECTIVES

- **Describe Kepler's laws of planetary motion.**
- **Relate Newton's mathematical analysis of gravitational force to the elliptical planetary orbits proposed by Kepler.**
- **Solve problems involving orbital speed and period.**

KEPLER'S LAWS

People have studied the motions of the planets since ancient times. Until the middle of the 16th century, most people believed that Earth was at the center of the universe. Originally, it was believed that the sun and other planets orbited Earth in perfect circles. However, this model did not account for all of the observations of planetary motion.

In the second century CE, Claudius Ptolemy developed an elaborate theory of planetary motion. Ptolemy's theory attempted to reconcile observation with theory and to keep Earth at the center of the universe. In this theory, planets travel in small circles called *epicycles* while simultaneously traveling in larger circular orbits. Even Ptolemy's complex model did not fully agree with observation, although the model did explain more than previous theories.

In 1543, the Polish astronomer Nicolaus Copernicus (1473–1543) published *On the Revolutions of the Heavenly Spheres,* in which he proposed that Earth and other planets orbit the sun in perfect circles. **Figure 11** shows a sun-centered planetary model that is believed to have been made for King George III of England. The idea of a sun-centered universe was not completely new in the 16th century. A Greek named Aristarchus theorized 1700 years before Copernicus did that Earth revolved around the sun, but most other scientists did not accept his theory.

Kepler's three laws describe the motion of the planets

The astronomer Tycho Brahe (1546–1601) made many precise observations of the planets and stars. However, some of Brahe's data did not agree with the Copernican model. The astronomer Johannes Kepler (1571–1630) worked for many years to reconcile Copernican theory with Brahe's data. Kepler's analysis led to three laws of planetary motion, which were developed a generation before Newton's law of universal gravitation. Kepler's three laws can be summarized as shown on the next page.

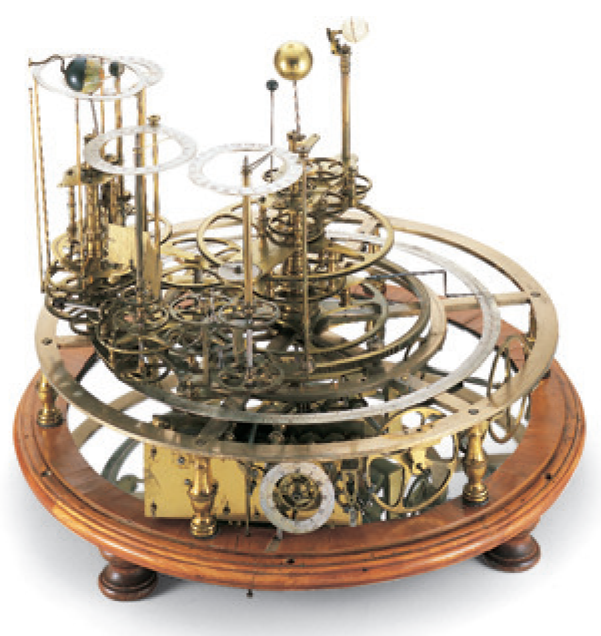


Figure 11
This elaborate planetary model—called an *orrery*—shows the motions of Mercury, Venus, and Earth around the sun. The model also shows the moon's inclined orbit around Earth.

KEPLER'S LAWS OF PLANETARY MOTION

First Law: Each planet travels in an elliptical orbit around the sun, and the sun is at one of the focal points.

Second Law: An imaginary line drawn from the sun to any planet sweeps out equal areas in equal time intervals.

Third Law: The square of a planet's orbital period (T^2) is proportional to the cube of the average distance (r^3) between the planet and the sun, or $T^2 \propto r^3$.

Kepler's first law states that the planets' orbits are ellipses rather than circles. Kepler discovered this law while working with Brahe's data for the orbit of Mars. While trying to explain the data, Kepler experimented with 70 different circular orbits and generated numerous pages of calculations. He finally realized that if the orbit is an ellipse rather than a circle and the sun is at one focal point of the ellipse, the data fit perfectly.

Kepler's second law states that an imaginary line from the sun to any planet sweeps out equal areas in equal times, as shown in **Figure 12.** In other words, if the time a planet takes to travel the arc on the left (Δt_1) is equal to the time the planet takes to cover the arc on the right (Δt_2), then the area A_1 is equal to the area A_2. Thus, the planets travel faster when they are closer to the sun.

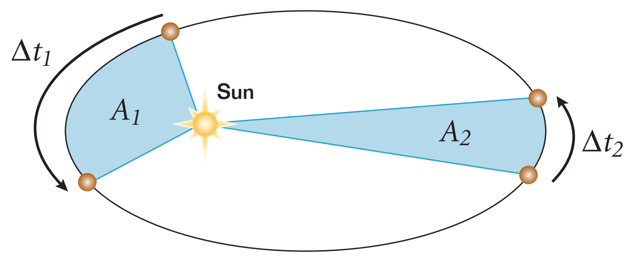


Figure 12
This diagram illustrates a planet moving in an elliptical orbit around the sun. If Δt_1 equals Δt_2, then the two shaded areas are equal. Thus, the planet travels faster when it is closer to the sun and slower when it is farther away.

While Kepler's first two laws describe the motion of each planet individually, his third law relates the orbital periods and distances of one planet to those of another planet. The orbital period (T) is the time a planet takes to finish one full revolution, and the distance (r) is the mean distance between the planet and the sun. Kepler's third law relates the orbital period and mean distance for two orbiting planets as follows:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}, \text{ or } T^2 \propto r^3$$

This law also applies to satellites orbiting Earth, including our moon. In that case, r is the distance between the orbiting satellite and Earth. The proportionality constant depends on the mass of the central object.

Kepler's laws are consistent with Newton's law of gravitation

Newton used Kepler's laws to support his law of gravitation. For example, Newton proved that if force is inversely proportional to distance squared, as stated in the law of universal gravitation, the resulting orbit must be an ellipse or a circle. He also demonstrated that his law of gravitation could be used to derive Kepler's third law. (Try a similar derivation yourself in the Quick Lab at right.) The fact that Kepler's laws closely matched observations gave additional support for Newton's theory of gravitation.

Quick Lab

Kepler's Third Law

You can mathematically show how Kepler's third law can be derived from Newton's law of universal gravitation (assuming circular orbits). To begin, recall that the centripetal force is provided by the gravitational force. Set the equations for gravitational and centripetal force equal to one another, and solve for v_t^2. Because speed equals distance divided by time and because the distance for one period is the circumference ($2\pi r$), $v_t = 2\pi r/T$. Square this value, substitute the squared value into your previous equation, and then isolate T^2. How does your result relate to Kepler's third law?

Did you know?

We generally speak of the moon orbiting Earth, but they are actually both in orbit around the center of mass of the Earth-moon system. Because Earth is so much more massive than the moon, their common center of mass lies inside Earth. Thus, the moon appears to orbit Earth. The center of mass does not always lie inside one of the bodies. For example, Pluto and its moon, Charon, orbit a center of mass that lies between them. Also, many binary star systems have two stars that orbit a common center of mass between the stars.

Kepler's third law describes orbital period

According to Kepler's third law, $T^2 \propto r^3$. The constant of proportionality between these two variables turns out to be $4\pi^2/Gm$, where m is the mass of the object being orbited. (To learn why this is the case, try the Quick Lab on the previous page.) Thus, Kepler's third law can also be stated as follows:

$$T^2 = \left(\frac{4\pi^2}{Gm}\right)r^3$$

The square root of the above equation, which is shown below on the left, describes the period of any object that is in a circular orbit. The speed of an object that is in a circular orbit depends on the same factors that the period does, as shown in the equation on the right. The assumption of a circular orbit provides a close approximation for real orbits in our solar system because all planets except Mercury and Pluto have orbits that are nearly circular.

PERIOD AND SPEED OF AN OBJECT IN CIRCULAR ORBIT

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} \qquad v_t = \sqrt{G\frac{m}{r}}$$

$$\text{orbital period} = 2\pi\sqrt{\frac{(\text{mean radius})^3}{(\text{constant})(\text{mass of central object})}}$$

$$\text{orbital speed} = \sqrt{(\text{constant})\left(\frac{\text{mass of central object}}{\text{mean radius}}\right)}$$

Note that m in both equations is the mass of the central object that is being orbited. The mass of the planet or satellite that is in orbit does not affect its speed or period. The mean radius (r) is the distance between the centers of the two bodies. For an artificial satellite orbiting Earth, r is equal to Earth's mean radius plus the satellite's distance from Earth's surface (its "altitude"). **Table 1** gives planetary data that can be used to calculate orbital speeds and periods.

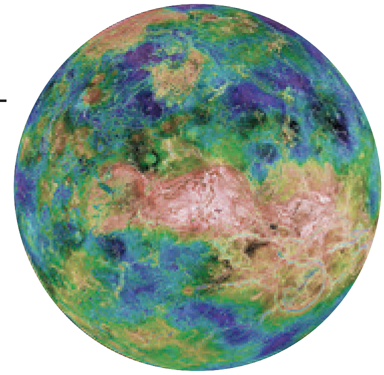
Table 1 Planetary Data

Planet	Mass (kg)	Mean radius (m)	Mean distance from sun (m)	Planet	Mass (kg)	Mean radius (m)	Mean distance from sun (m)
Earth	5.97×10^{24}	6.38×10^{6}	1.50×10^{11}	Neptune	1.02×10^{26}	2.48×10^{7}	4.50×10^{12}
Earth's moon	7.35×10^{22}	1.74×10^{6}	—	Pluto	1.25×10^{22}	1.20×10^{6}	5.87×10^{12}
Jupiter	1.90×10^{27}	7.15×10^{7}	7.79×10^{11}	Saturn	5.68×10^{26}	6.03×10^{7}	1.43×10^{12}
Mars	6.42×10^{23}	3.40×10^{6}	2.28×10^{11}	Sun	1.99×10^{30}	6.96×10^{8}	—
Mercury	3.30×10^{23}	2.44×10^{6}	5.79×10^{10}	Uranus	8.68×10^{25}	2.56×10^{7}	2.87×10^{12}
				Venus	4.87×10^{24}	6.05×10^{6}	1.08×10^{11}

SAMPLE PROBLEM D

Period and Speed of an Orbiting Object

PROBLEM



The color-enhanced image of Venus shown here was compiled from data taken by *Magellan*, the first planetary spacecraft to be launched from a space shuttle. During the spacecraft's fifth orbit around Venus, *Magellan* traveled at a mean altitude of 361 km. If the orbit had been circular, what would *Magellan*'s period and speed have been?

SOLUTION

1. DEFINE **Given:** $r_1 = 361 \text{ km} = 3.61 \times 10^5 \text{ m}$

Unknown: $T = ?$ $\quad v_t = ?$

2. PLAN **Choose an equation or situation:** Use the equations for the period and speed of an object in a circular orbit.

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} \qquad v_t = \sqrt{G\frac{m}{r}}$$

Use **Table 1** to find the values for the radius (r_2) and mass (m) of Venus.

$$r_2 = 6.05 \times 10^6 \text{ m} \qquad m = 4.87 \times 10^{24} \text{ kg}$$

Find r by adding the distance between the spacecraft and Venus's surface (r_1) to Venus's radius (r_2).

$$r = r_1 + r_2 = (3.61 \times 10^5 \text{ m}) + (6.05 \times 10^6 \text{ m}) = 6.41 \times 10^6 \text{ m}$$

3. CALCULATE **Substitute the values into the equations and solve:**

$$T = 2\pi\sqrt{\frac{(6.41 \times 10^6 \text{ m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(4.87 \times 10^{24} \text{ kg})}} = \boxed{5.66 \times 10^3 \text{ s}}$$

$$v_t = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left(\frac{4.87 \times 10^{24} \text{ kg}}{6.41 \times 10^6 \text{ m}}\right)} = \boxed{7.12 \times 10^3 \text{ m/s}}$$

4. EVALUATE *Magellan* takes $(5.66 \times 10^3 \text{ s})(1 \text{ min}/60 \text{ s}) \approx 94$ min to complete one orbit.

PRACTICE D

Period and Speed of an Orbiting Object

1. Find the orbital speed and period that the *Magellan* satellite from Sample Problem D would have at the same mean altitude above Earth, Jupiter, and Earth's moon.

2. At what distance above Earth would a satellite have a period of 125 min?

Quick Lab

Elevator Acceleration

MATERIALS LIST

- elevator
- bathroom scale
- watch or stopwatch

In this activity, you will stand on a bathroom scale while riding an elevator up to the top floor and then back. Stand on the scale in a first-floor elevator, and record your weight. As the elevator moves up, record the scale reading for every two-second interval. Repeat the process as the elevator moves down.

Now, find the net force for each time interval, and then use Newton's second law to calculate the elevator's acceleration for each interval. How does the acceleration change? How does the elevator's maximum acceleration compare with free-fall acceleration?

WEIGHT AND WEIGHTLESSNESS

In the chapter about forces, you learned that weight is the magnitude of the force due to gravity. When you step on a bathroom scale, it does not actually measure your weight. The scale measures the downward force exerted on it. When your weight is the only downward force acting on the scale, the scale reading equals your weight. If a friend pushes down on you while you are standing on the scale, the scale reading will go up. However, your weight has not changed; the scale reading equals your weight plus the extra applied force. Because of Newton's third law, the downward force you exert on the scale equals the upward force exerted on you by the scale (the normal force). Thus, the scale reading is equal to the normal force acting on you.

For example, imagine you are standing in an elevator, as illustrated in **Figure 13.** When the elevator is at rest, as in **Figure 13(a),** the magnitude of the normal force is equal to your weight. A scale in the elevator would record your weight. When the elevator begins accelerating downward, as in **Figure 13(b),** the normal force will be smaller. The scale would now record an amount that is less than your weight. If the elevator's acceleration were equal to free-fall acceleration, as shown in **Figure 13(c),** you would be falling at the same rate as the elevator and would not feel the force of the floor at all. In this case, the scale would read zero. You still have the same weight, but you and the elevator are both falling with free-fall acceleration. In other words, no normal force is acting on you. This situation is called *apparent weightlessness.*

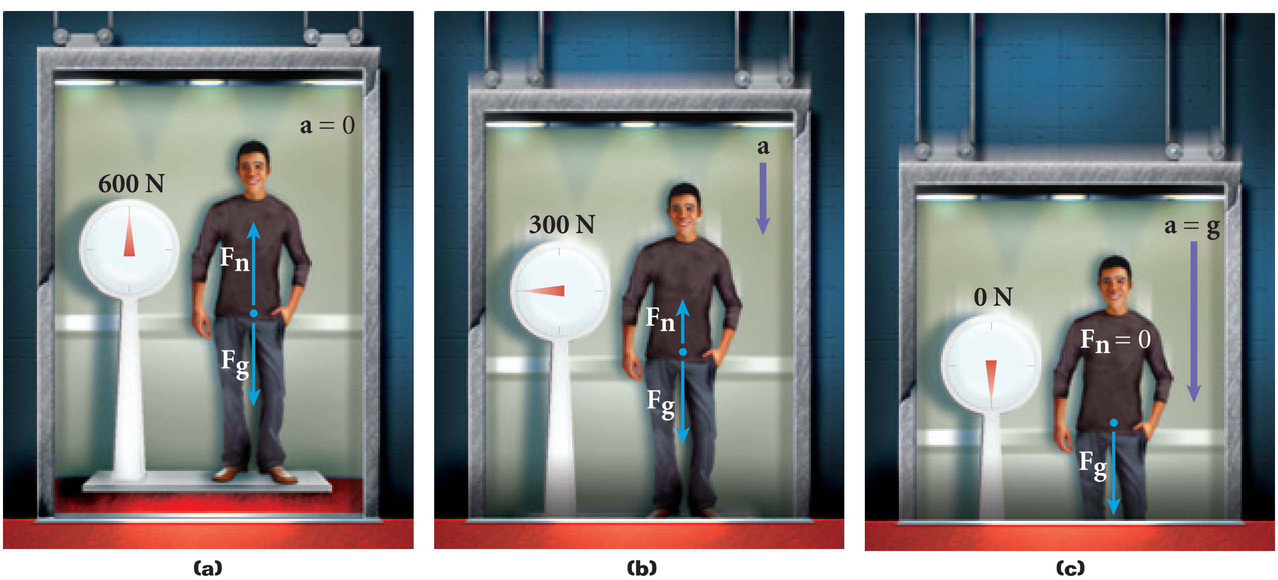


Figure 13
When this elevator accelerates, the normal force acting on the person changes. If the elevator were in free fall, the normal force would drop to zero and the person would experience a sensation of apparent weightlessness.

Astronauts in orbit experience apparent weightlessness

Astronauts floating in a space shuttle are experiencing apparent weightlessness. Because the shuttle is accelerating at the same rate as the astronauts are, this example is similar to the elevator in **Figure 13(c).** The force due to gravity keeps the astronauts and shuttle in orbit, but the astronauts *feel* weightless because no normal force is acting on them.

The human body relies on gravitational force. For example, this force pulls blood downward so that the blood collects in the veins of your legs when you are standing. Because the body of an astronaut in orbit accelerates along with the space shuttle, gravitational force has no effect on the body. This state can initially cause nausea and dizziness. Over time, it can pose serious health risks, such as weakened muscles and brittle bones. When astronauts return to Earth, their bodies need time to readjust to the effects of the gravitational force.

So far, we have been describing apparent weightlessness. Actual weightlessness occurs only in deep space, far from stars and planets. Gravitational force is never entirely absent, but it can become negligible at distances that are far enough away from any masses. In this case, a star or astronaut would not be pulled into an orbit but would instead drift in a straight line at constant speed.

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SECTION REVIEW

1. Compare Ptolemy's model of the solar system with Copernicus's. How does Kepler's first law of planetary motion refine Copernicus's model?
2. Does a planet in orbit around the sun travel at a constant speed? How do you know?
3. Suppose you know the mean distance between both Mercury and the sun and Venus and the sun. You also know the period of Venus's orbit around the sun. How can you find the period of Mercury's orbit?
4. Explain how Kepler's laws of planetary motion relate to Newton's law of universal gravitation.
5. Find the orbital speed and period of Earth's moon. The average distance between the centers of Earth and of the moon is 3.84×10^8 m.
6. **Critical Thinking** An amusement park ride raises people high into the air, suspends them for a moment, and then drops them at the rate of free-fall acceleration. Is a person in this ride experiencing apparent weightlessness, true weightlessness, or neither? Explain.
7. **Critical Thinking** Suppose you went on the ride described in item 6, held a penny in front of you, and released the penny at the moment the ride started to drop. What would you observe?

SECTION 4

Torque and Simple Machines

SECTION OBJECTIVES

- **Distinguish between torque and force.**
- **Calculate the magnitude of a torque on an object.**
- **Identify the six types of simple machines.**
- **Calculate the mechanical advantage of a simple machine.**

ROTATIONAL MOTION

Earlier in this chapter, you studied various examples of uniform circular motion, such as a spinning Ferris wheel or an orbiting satellite. During uniform circular motion, an object moves in a circular path and at constant speed. An object that is in circular motion is accelerating because the direction of the object's velocity is constantly changing. This centripetal acceleration is directed toward the center of the circle. The net force causing the acceleration is a centripetal force, which is also directed toward the center of the circle.

In this section, we will examine a related type of motion: the motion of a rotating rigid object. For example, consider a football that is spinning as it flies through the air. If gravity is the only force acting on the football, the football spins around a point called its *center of mass.* As the football moves through the air, its center of mass follows a parabolic path. Note that the center of mass is not always at the center of the object.

ADVANCED TOPICS

See "Rotation and Inertia" and "Rotational Dynamics" in **Appendix J: Advanced Topics** to learn more about rotational motion.

Rotational and translational motion can be separated

Imagine that you roll a strike while bowling. When the bowling ball strikes the pins, as shown in **Figure 14,** the pins spin in the air as they fly backward. Thus, they have both rotational and linear motion. These types of motion can be analyzed separately. In this section, we will isolate rotational motion. In particular, we will explore how to measure the ability of a force to rotate an object.

Figure 14
Pins that are spinning and flying through the air exhibit both rotational and translational motion.

THE MAGNITUDE OF A TORQUE

Imagine a cat trying to leave a house by pushing perpendicularly on a cat-flap door. **Figure 15** shows a cat-flap door hinged at the top. In this configuration, the door is free to rotate around a line that passes through the hinge. This is the door's *axis of rotation*. When the cat pushes at the outer edge of the door with a force that is perpendicular to the door, the door opens. The ability of a force to rotate an object around some axis is measured by a quantity called **torque.**

torque

a quantity that measures the ability of a force to rotate an object around some axis

Torque depends on the force and the lever arm

If a cat pushed on the door with the same force but at a point closer to the hinge, the door would be more difficult to rotate. How easily an object rotates depends not only on how much force is applied but also on where the force is applied. The farther the force is from the axis of rotation, the easier it is to rotate the object and the more torque is produced. The perpendicular distance from the axis of rotation to a line drawn along the direction of the force is called the **lever arm.**

lever arm

the perpendicular distance from the axis of rotation to a line drawn along the direction of the force

Figure 16 shows a diagram of the force **F** applied by the pet perpendicular to the cat-flap door. If you examine the definition of *lever arm*, you will see that in this case the lever arm is the distance d shown in the figure, the distance from the pet's nose to the hinge. That is, d is the perpendicular distance from the axis of rotation to the line along which the applied force acts. If the pet pressed on the door at a higher point, the lever arm would be shorter. As a result, the cat would need to exert a greater force to apply the same torque.

Figure 15
The cat-flap door rotates on a hinge, allowing pets to enter and leave a house at will.

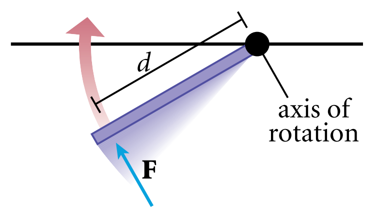


Figure 16
A force applied to an extended object can produce a torque. This torque, in turn, causes the object to rotate.

Quick Lab

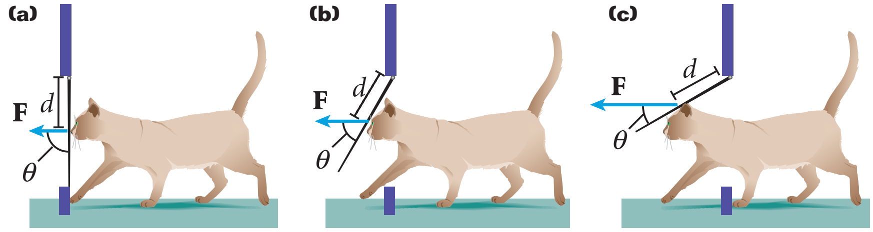
Changing the Lever Arm

MATERIALS LIST

- door
- masking tape

In this activity, you will explore how the amount of force required to open a door changes when the lever arm changes. Using only perpendicular forces, open a door several times by applying a force at different distances from the hinge. You may have to tape the latch so that the door will open when you push without turning the knob. Because the angle of the applied force is kept constant, decreasing the distance to the hinge decreases the lever arm. Compare the relative effort required to open the door when pushing near the edge to that required when pushing near the hinged side of the door. Summarize your findings in terms of torque and the lever arm.

Figure 17
In each example, the cat is pushing on the door at the same distance from the axis. To produce the same torque, the cat must apply greater force for smaller angles.



The lever arm depends on the angle

Forces do not have to be perpendicular to an object to cause the object to rotate. Imagine the cat-flap door again. In **Figure 17(a),** the force exerted by the cat is perpendicular. When the angle is less than 90°, as in **(b)** and **(c),** the door will still rotate, but not as easily. The symbol for torque is the Greek letter *tau* (τ), and the magnitude of the torque is given by the following equation:

TORQUE

$$\tau = Fd \sin \theta$$

torque = force × lever arm

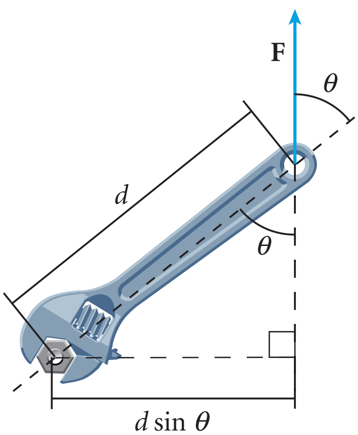


Figure 18
The direction of the lever arm is always perpendicular to the direction of the applied force.

The SI unit of torque is the N•m. Notice that the inclusion of the factor sin θ in this equation takes into account the changes in torque shown in **Figure 17.**

Figure 18 shows a wrench pivoted around a bolt. In this case, the applied force acts at an angle to the wrench. The quantity d is the distance from the axis of rotation to the point where force is applied. The quantity *d sin θ*, however, is the *perpendicular* distance from the axis of rotation to a line drawn along the direction of the force. *Thus, d sin θ is the lever arm.* Note that the perpendicular distance between the door hinge and the point of application of force **F** in **Figure 17** decreases as the cat goes further through the door.

THE SIGN OF A TORQUE

Torque, like displacement and force, is a vector quantity. In this textbook, we will assign each torque a positive or negative sign, depending on the direction the force tends to rotate an object. We will use the convention that the sign of the torque resulting from a force is positive if the rotation is counterclockwise and negative if the rotation is clockwise. In calculations, remember to assign positive and negative values to forces and displacements according to the sign convention established in the chapter "Motion in One Dimension."

PHYSICS INTERACTIVE TUTOR
Module 9
"Torque"
provides an interactive lesson with guided problem-solving practice to teach you about many aspects of rotational motion, including torque.

TIP
To determine the sign of a torque, imagine that the torque is the only one acting on the object and that the object is free to rotate. Visualize the direction that the object would rotate. If more than one force is acting, treat each force separately. Be careful to associate the correct sign with each torque.

For example, imagine that you are pulling on a wishbone with a perpendicular force F_1 and that a friend is pulling in the opposite direction with a force F_2. If you pull the wishbone so that it would rotate counterclockwise, then you exert a positive torque of magnitude F_1d_1. Your friend, on the other hand, exerts a negative torque, $-F_2d_2$. To find the net torque acting on the wishbone, simply add up the individual torques.

$$\tau_{net} = \Sigma\tau = \tau_1 + \tau_2 = F_1d_1 + (-F_2d_2)$$

When you properly apply the sign convention, the sign of the net torque will tell you which way the object will rotate, if at all.

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For a variety of links related to this chapter, go to www.scilinks.org

Topic: Torque
SciLinks Code: HF61538

SAMPLE PROBLEM E

Torque

PROBLEM

A basketball is being pushed by two players during tip-off. One player exerts an upward force of 15 N at a perpendicular distance of 14 cm from the axis of rotation. The second player applies a downward force of 11 N at a perpendicular distance of 7.0 cm from the axis of rotation. Find the net torque acting on the ball about its center of mass.

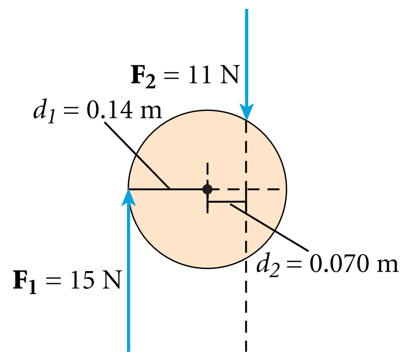
SOLUTION

1. DEFINE

Given: $F_1 = 15$ N $\quad F_2 = 11$ N
$d_1 = 0.14$ m $\quad d_2 = 0.070$ m

Unknown: $\tau_{net} = ?$

Diagram:



2. PLAN

Choose an equation or situation: Apply the definition of torque to each force, and add up the individual torques.

$$\tau = Fd$$
$$\tau_{net} = \tau_1 + \tau_2 = F_1d_1 + F_2d_2$$

TIP *The factor sin θ is not included because each given distance is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force.*

3. CALCULATE

Substitute the values into the equations and solve: First, determine the torque produced by each force. Use the standard convention for signs.

$$\tau_1 = F_1d_1 = (15\ \text{N})(-0.14\ \text{m}) = -2.1\ \text{N}\bullet\text{m}$$

$$\tau_2 = F_2d_2 = (-11\ \text{N})(0.070\ \text{m}) = -0.77\ \text{N}\bullet\text{m}$$

$$\tau_{net} = -2.1\ \text{N}\bullet\text{m} - 0.77\ \text{N}\bullet\text{m}$$

$$\boxed{\tau_{net} = -2.9\ \text{N}\bullet\text{m}}$$

4. EVALUATE

The net torque is negative, so the ball rotates in a clockwise direction.

PRACTICE E

Torque

1. Find the magnitude of the torque produced by a 3.0 N force applied to a door at a perpendicular distance of 0.25 m from the hinge.

2. A simple pendulum consists of a 3.0 kg point mass hanging at the end of a 2.0 m long light string that is connected to a pivot point.

a. Calculate the magnitude of the torque (due to gravitational force) around this pivot point when the string makes a 5.0° angle with the vertical.

b. Repeat this calculation for an angle of 15.0°.

3. If the torque required to loosen a nut on the wheel of a car has a magnitude of 40.0 N•m, what *minimum* force must be exerted by a mechanic at the end of a 30.0 cm wrench to loosen the nut?

TYPES OF SIMPLE MACHINES

What do you do when you need to pry a cap off a bottle of soda? You probably use a bottle opener, as shown in **Figure 19.** Similarly, you would probably use scissors to cut paper or a hammer to drive a nail into a board. All of these devices make your task easier. These devices are all examples of *machines.*

The term *machine* may bring to mind intricate systems with multicolored wires and complex gear-and-pulley systems. Compared with internal-combustion engines or airplanes, simple devices such as hammers, scissors, and bottle openers may not seem like machines, but they are.

A machine is any device that transmits or modifies force, usually by changing the force applied to an object. All machines are combinations or modifications of six fundamental types of machines, called *simple machines.* These six simple machines are the lever, pulley, inclined plane, wheel and axle, wedge, and screw, as shown in **Table 2** on the next page.

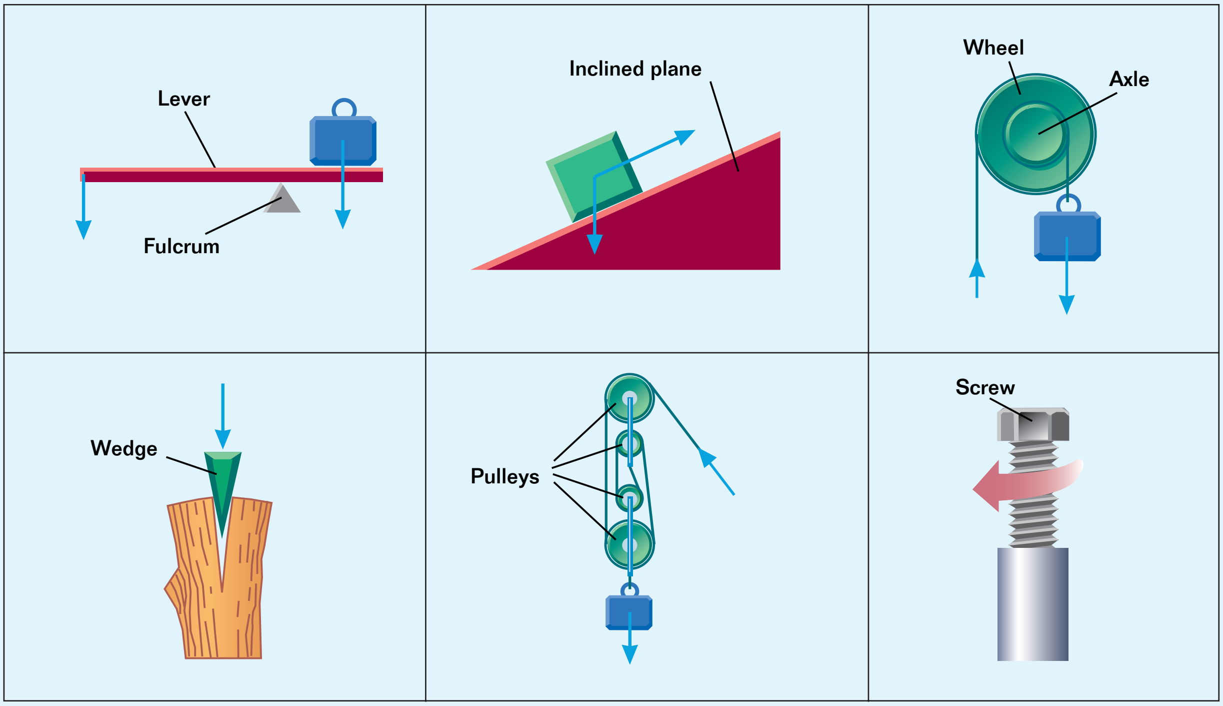
Figure 19
Because this bottle opener makes work easier, it is an example of a machine.

Using simple machines

Because the purpose of a simple machine is to change the direction or magnitude of an input force, a useful way of characterizing a simple machine is to compare how large the output force is relative to the input force. This ratio, called the machine's *mechanical advantage,* is written as follows:

$$MA = \frac{\text{output force}}{\text{input force}} = \frac{F_{out}}{F_{in}}$$

Table 2 Six Simple Machines



One example of mechanical advantage is the use of the back of a hammer to pry a nail from a board. In this example, the hammer is a type of lever. A person applies an input force to one end of the handle. The handle, in turn, exerts an output force on the head of a nail stuck in a board. If friction is disregarded, the input torque will equal the output torque. This relation can be written as follows:

$$\tau_{in} = \tau_{out}$$
$$F_{in}d_{in} = F_{out}d_{out}$$

Substituting this expression into the definition of mechanical advantage gives the following result:

$$MA = \frac{F_{out}}{F_{in}} = \frac{d_{in}}{d_{out}}$$

The longer the input lever arm as compared with the output lever arm, the greater the mechanical advantage is. This in turn indicates the factor by which the input force is amplified. If the force of the board on the nail is 99 N and if the mechanical advantage is 10, then an input force of 10 N is enough to pull out the nail. Without a machine, the nail could not be removed unless the input force was greater than 99 N.

TIP *This equation can be used to predict the output force for a given input force if there is no friction. The equation is not valid if friction is taken into account. With friction, the output force will be less than expected, and thus* $\frac{d_{in}}{d_{out}}$ *will not equal* $\frac{F_{out}}{F_{in}}$.

Figure 20
Lifting this trunk directly up requires more force than pushing it up the ramp, but the same amount of work is done in both cases.

Machines can alter the force and the distance moved

You have learned that mechanical energy is conserved in the absence of friction. This law holds for machines as well. A machine can increase (or decrease) the force acting on an object at the expense (or gain) of the distance moved, but the product of the two—the work done on the object—is constant.

For example, **Figure 20** shows two examples of a trunk being loaded onto a truck. **Figure 21** illustrates both examples schematically. In one example, the trunk is lifted directly onto the truck. In the other example, the trunk is pushed up an incline into the truck.

In the first example, a force ($\mathbf{F_1}$) of 360 N is required to lift the trunk, which moves through a distance (d_1) of 1.0 m. This requires 360 N•m of work (360 N × 1 m). In the second example, a lesser force ($\mathbf{F_2}$) of only 120 N would be needed (ignoring friction), but the trunk must be pushed a greater distance (d_2) of 3.0 m. This also requires 360 N•m of work (120 N × 3 m). As a result, the two methods require the same amount of energy.

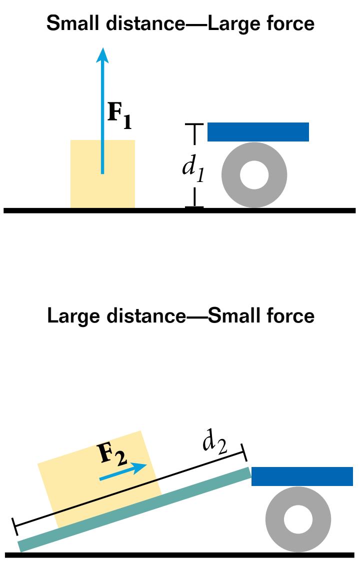


Figure 21
Simple machines can alter both the force needed to perform a task and the distance through which the force acts.

Efficiency is a measure of how well a machine works

The simple machines we have considered so far are ideal, frictionless machines. Real machines, however, are not frictionless. They dissipate energy. When the parts of a machine move and contact other objects, some of the input energy is dissipated as sound or heat. The *efficiency* of a machine is the ratio of useful work output to work input. It is defined by the following equation:

$$eff = \frac{W_{out}}{W_{in}}$$

If a machine is frictionless, then mechanical energy is conserved. This means that the work done on the machine (input work) is equal to the work done by the machine (output work) because work is a measure of energy transfer. Thus, the mechanical efficiency of an ideal machine is 1, or 100 percent. This is the best efficiency a machine can have. Because all real machines have at least a little friction, the efficiency of real machines is always less than 1.

SECTION REVIEW

1. Determine whether each of the following situations involves linear motion, rotational motion, or a combination of the two.
   a. a baseball dropped from the roof of a house
   b. a baseball rolling toward third base
   c. a pinwheel in the wind
   d. a door swinging open

2. What quantity describes the ability of a force to rotate an object? How does it differ from a force? On what quantities does it depend?

3. How would the force needed to open a door change if you put the handle in the middle of the door?

4. What are three ways that a cat pushing on a cat-flap door can change the amount of torque applied to the door?

5. The efficiency of a squeaky pulley system is 73 percent. The pulleys are used to raise a mass to a certain height. What force is exerted on the machine if a rope is pulled 18.0 m in order to raise a 58 kg mass a height of 3.0 m?

6. A person lifts a 950 N box by pushing it up an incline. If the person exerts a force of 350 N along the incline, what is the mechanical advantage of the incline?

7. You are attempting to move a large rock by using a long lever. Will the work you do on the lever be greater than, the same as, or less than the work done by the lever on the rock? Explain.

8. **Interpreting Graphics** Calculate the torque for each force acting on the bar in **Figure 22.** Assume the axis is perpendicular to the page and passes through point *O*. In what direction will the object rotate?

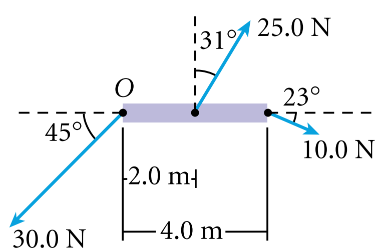


Figure 22

9. **Interpreting Graphics** **Figure 23** shows an example of a Rube Goldberg machine. Identify two types of simple machines that are included in this compound machine.

10. **Critical Thinking** A bicycle can be described as a combination of simple machines. Identify two types of simple machines that are used to propel a typical bicycle.

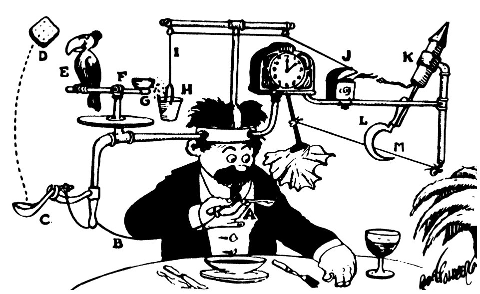


Figure 23

CHAPTER 7

Highlights

KEY TERMS

centripetal acceleration (p. 235)

gravitational force (p. 240)

torque (p. 255)

lever arm (p. 255)

PROBLEM SOLVING

See **Appendix D: Equations** for a summary of the equations introduced in this chapter. If you need more problem-solving practice, see **Appendix I: Additional Problems.**

KEY IDEAS

Section 1 Circular Motion

- An object that revolves about a single axis undergoes circular motion.
- An object in circular motion has a centripetal acceleration and a centripetal force, which are both directed toward the center of the circular path.

Section 2 Newton's Law of Universal Gravitation

- Every particle in the universe is attracted to every other particle by a force that is directly proportional to the product of the particles' masses and inversely proportional to the square of the distance between the particles.
- Gravitational field strength is the gravitational force that would be exerted on a unit mass at any given point in space and is equal to free-fall acceleration.

Section 3 Motion in Space

- Kepler developed three laws of planetary motion.
- Both the period and speed of an object that is in a circular orbit around another object depend on two quantities: the mass of the central object and the distance between the centers of the objects.

Section 4 Torque and Simple Machines

- Torque is a measure of a force's ability to rotate an object.
- The torque on an object depends on the magnitude of the applied force and on the lever arm.
- Simple machines provide a mechanical advantage.

Variable Symbols

Quantities		Units	
v_t	tangential speed	m/s	meters/second
a_c	centripetal acceleration	m/s^2	meters/second^2
F_c	centripetal force	N	newtons
F_g	gravitational force	N	newtons
g	gravitational field strength	N/kg	newtons/kilogram
T	orbital period	s	seconds
τ	torque	N•m	newton meter

CHAPTER 7 Review

CIRCULAR MOTION

Review Questions

1. When a solid wheel rotates about a fixed axis, do all of the points of the wheel have the same tangential speed?

2. Correct the following statement: The racing car rounds the turn at a constant velocity of 145 km/h.

3. Describe the path of a moving body whose acceleration is constant in magnitude at all times and is perpendicular to the velocity.

4. Give an example of a situation in which an automobile driver can have a centripetal acceleration but no tangential acceleration.

Conceptual Questions

5. The force exerted by a spring increases as the spring stretches. Imagine that you attach a heavy object to one end of a spring and then, while holding the spring's other end, whirl the spring and object in a horizontal circle. Does the spring stretch? Explain.

6. Can a car move around a circular racetrack so that the car has a tangential acceleration but no centripetal acceleration?

7. Why does mud fly off a rapidly turning wheel?

Practice Problems

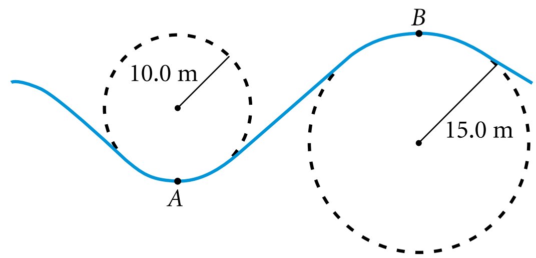
For problems 8–9, see Sample Problem A.

8. A building superintendent twirls a set of keys in a circle at the end of a cord. If the keys have a centripetal acceleration of 145 m/s^2 and the cord has a length of 0.34 m, what is the tangential speed of the keys?

9. A sock stuck to the side of a clothes-dryer barrel has a centripetal acceleration of 28 m/s^2. If the dryer barrel has a radius of 27 cm, what is the tangential speed of the sock?

For problems 10–11, see Sample Problem B.

10. A roller-coaster car speeds down a hill past point *A* and then rolls up a hill past point *B*, as shown below.
    - **a.** The car has a speed of 20.0 m/s at point *A*. If the track exerts a normal force on the car of 2.06×10^4 N at this point, what is the mass of the car? (Be sure to account for gravitational force.)
    - **b.** What is the maximum speed the car can have at point *B* for the gravitational force to hold it on the track?



11. Tarzan tries to cross a river by swinging from one bank to the other on a vine that is 10.0 m long. His speed at the bottom of the swing is 8.0 m/s. Tarzan does not know that the vine has a breaking strength of 1.0×10^3 N. What is the largest mass that Tarzan can have and still make it safely across the river?

NEWTON'S LAW OF UNIVERSAL GRAVITATION

Review Questions

12. Identify the influence of mass and distance on gravitational forces.

13. If a satellite orbiting Earth is in free fall, why does the satellite not fall and crash into Earth?

14. How does the gravitational force exerted by Earth on the sun compare with the gravitational force exerted by the sun on Earth?

15. What simple observation confirms that gravitational mass and inertial mass are equal?

Conceptual Questions

16. Would you expect tides to be higher at the equator or at the North Pole? Why?

17. Given Earth's radius, how could you use the value of *G* to calculate Earth's mass?

Practice Problems

For problems 18–19, see Sample Problem C.

18. The gravitational force of attraction between two students sitting at their desks in physics class is 3.20×10^{-8} N. If one student has a mass of 50.0 kg and the other has a mass of 60.0 kg, how far apart are the students sitting?

19. If the gravitational force between the electron (9.11×10^{-31} kg) and the proton (1.67×10^{-27} kg) in a hydrogen atom is 1.0×10^{-47} N, how far apart are the two particles?

MOTION IN SPACE

Review Questions

20. Compare and contrast Kepler's model of the solar system with Copernicus's model.

21. How do Kepler's laws help support Newton's theory of gravitation?

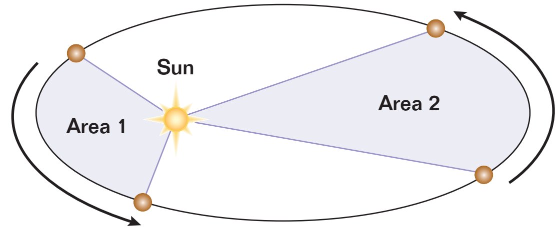
22. You are standing on a scale in an elevator. For a brief time, the elevator descends with free-fall acceleration. What does the scale show your weight to be during that time interval?

23. Astronauts floating around inside the space shuttle are not actually in a zero-gravity environment. What is the real reason astronauts seem weightless?

Conceptual Questions

24. A tiny alien spaceship (m = 0.25 kg) and the *International Space Station* are both orbiting Earth in circular orbits and at the same distance from Earth. Which one has a greater orbital speed?

25. The planet shown below sweeps out Area 1 in half the time that the planet sweeps out Area 2. How much bigger is Area 2 than Area 1?



26. Comment on the statement, "There is no gravity in outer space."

Practice Problems

For problems 27–29, see Sample Problem D.

27. What would be the orbital speed and period of a satellite in orbit 1.44×10^{8} m above Earth?

28. A satellite with an orbital period of exactly 24.0 h is always positioned over the same spot on Earth. This is known as a *geosynchronous* orbit. Television, communication, and weather satellites use geosynchronous orbits. At what distance would a satellite have to orbit Earth in order to have a geosynchronous orbit?

29. The distance between the centers of a small moon and a planet in our solar system is 2.0×10^{8} m. If the moon's orbital period is 5.0×10^{4} s, what is the planet? (See **Table 1** of the chapter for planet masses.)

TORQUE AND SIMPLE MACHINES

Review Questions

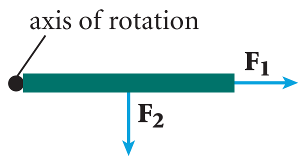
30. Why is it easier to loosen the lid from the top of a paint can with a long-handled screwdriver than with a short-handled screwdriver?

31. If a machine cannot multiply the amount of work, what is the advantage of using such a machine?

32. In the equation for the magnitude of a torque, what does the quantity $d \sin \theta$ represent?

Conceptual Questions

33. Which of the forces acting on the rod shown below will produce a torque about the axis at the left end of the rod?



34. Two forces equal in magnitude but opposite in direction act at the same point on an object. Is it possible for there to be a net torque on the object? Explain.

35. You are attempting to move a large rock by using a long lever. Is it more effective to place the lever's axis of rotation nearer to your hands or nearer to the rock? Explain.

36. A perpetual motion machine is a machine that, when set in motion, will never come to a halt. Why is such a machine not possible?

Practice Problems

For problems 37–38, see Sample Problem E.

37. A bucket filled with water has a mass of 54 kg and is hanging from a rope that is wound around a 0.050 m radius stationary cylinder. If the cylinder does not rotate and the bucket hangs straight down, what is the magnitude of the torque the bucket produces around the center of the cylinder?

38. A mechanic jacks up the front of a car to an angle of 8.0° with the horizontal in order to change the front tires. The car is 3.05 m long and has a mass of 1130 kg. Gravitational force acts at the center of mass, which is located 1.12 m from the front end. The rear wheels are 0.40 m from the back end. Calculate the magnitude of the torque exerted by the jack.

MIXED REVIEW

39. A 2.00×10^3 kg car rounds a circular turn of radius 20.0 m. If the road is flat and the coefficient of static friction between the tires and the road is 0.70, how fast can the car go without skidding?

40. During a solar eclipse, the moon, Earth, and sun lie on the same line, with the moon between Earth and the sun. What force is exerted on

a. the moon by the sun?
b. the moon by Earth?
c. Earth by the sun?

(See the table in Appendix F for data on the sun, moon, and Earth.)

41. A wooden bucket filled with water has a mass of 75 kg and is attached to a rope that is wound around a cylinder with a radius of 0.075 m. A crank with a turning radius of 0.25 m is attached to the end of the cylinder. What minimum force directed perpendicularly to the crank handle is required to raise the bucket?

42. If the torque required to loosen a nut that holds a wheel on a car has a magnitude of 58 N•m, what force must be exerted at the end of a 0.35 m lug wrench to loosen the nut when the angle is 56°? (Hint: See **Figure 18** for an example, and assume that θ is 56°.)

43. In a canyon between two mountains, a spherical boulder with a radius of 1.4 m is just set in motion by a force of 1600 N. The force is applied at an angle of 53.5° measured with respect to the vertical radius of the boulder. What is the magnitude of the torque on the boulder?

44. The hands of the clock in the famous Parliament Clock Tower in London are 2.7 m and 4.5 m long and have masses of 60.0 kg and 100.0 kg, respectively. Calculate the torque around the center of the clock due to the weight of these hands at 5:20. The weight of each hand acts at the center of mass (the midpoint of the hand).

45. The efficiency of a pulley system is 64 percent. The pulleys are used to raise a mass of 78 kg to a height of 4.0 m. What force is exerted on the rope of the pulley system if the rope is pulled for 24 m in order to raise the mass to the required height?

46. A crate is pulled 2.0 m at constant velocity along a 15° incline. The coefficient of kinetic friction between the crate and the plane is 0.160. Calculate the efficiency of this procedure.

47. A pulley system is used to lift a piano 3.0 m. If a force of 2200 N is applied to the rope as the rope is pulled in 14 m, what is the efficiency of the machine? Assume the mass of the piano is 750 kg.

48. A pulley system has an efficiency of 87.5 percent. How much of the rope must be pulled in if a force of 648 N is needed to lift a 150 kg desk 2.46 m? (Disregard friction.)

49. Jupiter's four large moons—Io, Europa, Ganymede, and Callisto—were discovered by Galileo in 1610. Jupiter also has dozens of smaller moons. Jupiter's rocky, volcanically-active moon Io is about the size of Earth's moon. Io has radius of about 1.82×10^6 m, and the mean distance between Io and Jupiter is 4.22×10^8 m.

a. If Io's orbit were circular, how many days would it take for Io to complete one full revolution around Jupiter?

b. If Io's orbit were circular, what would its orbital speed be?

Graphing Calculator  Practice

Torque

One of the terms introduced in this chapter is *torque*. Torque is a measure of the ability of a force to rotate an object around an axis. As you learned earlier in this chapter, torque is described by the following equation:

$$\tau = Fd\sin\theta$$

In this equation, F is the applied force, d is the distance from the axis of rotation, and θ is the angle at which the force is applied. A mechanic using a long wrench to loosen a "frozen" bolt is a common illustration of this equation.

In this graphing calculator activity, you will determine how torque relates to the angle of the applied force and to the distance of application.

Visit go.hrw.com and type in the keyword **HF6CMGX** to find this graphing calculator activity. Refer to **Appendix B** for instructions on downloading the program for this activity.

50. A 13 500 N car traveling at 50.0 km/h rounds a curve of radius 2.00×10^2 m. Find the following:

a. the centripetal acceleration of the car

b. the centripetal force

c. the minimum coefficient of static friction between the tires and the road that will allow the car to round the curve safely

51. The arm of a crane at a construction site is 15.0 m long, and it makes an angle of 20.0° with the horizontal. Assume that the maximum load the crane can handle is limited by the amount of torque the load produces around the base of the arm.

a. What is the magnitude of the maximum torque the crane can withstand if the maximum load the crane can handle is 450 N?

b. What is the maximum load for this crane at an angle of 40.0° with the horizontal?

52. At the sun's surface, the gravitational force between the sun and a 5.00 kg mass of hot gas has a magnitude of 1370 N. Assuming that the sun is spherical, what is the sun's mean radius?

53. An automobile with a tangential speed of 55.0 km/h follows a circular road that has a radius of 40.0 m. The automobile has a mass of 1350 kg. The pavement is wet and oily, so the coefficient of kinetic friction between the car's tires and the pavement is only 0.500. How large is the available frictional force? Is this frictional force large enough to maintain the automobile's circular motion?

54. A force is applied to a door at an angle of 60.0° and 0.35 m from the hinge. What force produces a torque with a magnitude of 2.0 N•m? How large is the maximum torque this force can exert?

55. Imagine a balance with unequal arms. An earring placed in the left basket was balanced by 5.00 g of standard masses on the right. When placed in the right basket, the same earring required 15.00 g on the left to balance. Which was the longer arm? Do you need to know the exact length of each arm to determine the mass of the earring? Explain.

Alternative Assessment

1. Research the historical development of the concept of gravitational force. Find out how scientists' ideas about gravity have changed over time. Identify the contributions of different scientists, such as Galileo, Kepler, Newton, and Einstein. How did each scientist's work build on the work of earlier scientists? Analyze, review, and critique the different scientific explanations of gravity. Focus on each scientist's hypotheses and theories. What are their strengths? What are their weaknesses? What do scientists think about gravity now? Use scientific evidence and other information to support your answers. Write a report or prepare an oral presentation to share your conclusions.

2. Describe exactly which measurements you would need to make in order to identify the torques at work during a ride on a specific bicycle. Your plans should include measurements you can make with equipment available to you. If others in the class analyzed different bicycle models, compare the models for efficiency and mechanical advantage.

3. Prepare a poster or a series of models of simple machines, explaining their use and how they work. Include a schematic diagram next to each sample or picture to identify the fulcrum, lever arm, and resistance. Add your own examples to the following list: nail clipper, wheelbarrow, can opener, nutcracker, electric drill, screwdriver, tweezers, and key in lock.

Standardized Test Prep

MULTIPLE CHOICE

1. An object moves in a circle at a constant speed. Which of the following is *not* true of the object?
   A. Its centripetal acceleration points toward the center of the circle.
   B. Its tangential speed is constant.
   C. Its velocity is constant.
   D. A centripetal force acts on the object.

Use the passage below to answer questions 2–3.

A car traveling at 15 m/s on a flat surface turns in a circle with a radius of 25 m.

2. What is the centripetal acceleration of the car?
   F. 2.4×10^{-2} m/s^2
   G. 0.60 m/s^2
   H. 9.0 m/s^2
   J. zero

3. What is the most direct cause of the car's centripetal acceleration?
   A. the torque on the steering wheel
   B. the torque on the tires of the car
   C. the force of friction between the tires and the road
   D. the normal force between the tires and the road

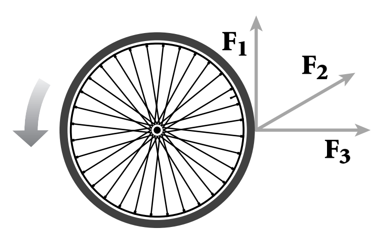
4. Earth ($m = 5.97 \times 10^{24}$ kg) orbits the sun ($m = 1.99 \times 10^{30}$ kg) at a mean distance of 1.50×10^{11} m. What is the gravitational force of the sun on Earth? ($G = 6.673 \times 10^{-11}$ N•m^2/kg^2)
   F. 5.29×10^{32} N
   G. 3.52×10^{22} N
   H. 5.90×10^{-2} N
   J. 1.77×10^{-8} N

5. Which of the following is a correct interpretation of the expression $a_g = g = G\frac{m_E}{r^2}$?
   A. Gravitational field strength changes with an object's distance from Earth.
   B. Free-fall acceleration changes with an object's distance from Earth.
   C. Free-fall acceleration is independent of the falling object's mass.
   D. All of the above are correct interpretations.

6. What data do you need to calculate the orbital speed of a satellite?
   F. mass of satellite, mass of planet, radius of orbit
   G. mass of satellite, radius of planet, area of orbit
   H. mass of satellite and radius of orbit only
   J. mass of planet and radius of orbit only

7. Which of the following choices correctly describes the orbital relationship between Earth and the sun?
   A. The sun orbits Earth in a perfect circle.
   B. Earth orbits the sun in a perfect circle.
   C. The sun orbits Earth in an ellipse, with Earth at one focus.
   D. Earth orbits the sun in an ellipse, with the sun at one focus.

Use the diagram below to answer questions 8–9.



8. The three forces acting on the wheel above have equal magnitudes. Which force will produce the greatest torque on the wheel?
   F. $\mathbf{F_1}$
   G. $\mathbf{F_2}$
   H. $\mathbf{F_3}$
   J. Each force will produce the same torque.

9. If each force is 6.0 N, the angle between $\mathbf{F_1}$ and $\mathbf{F_2}$ is 60.0°, and the radius of the wheel is 1.0 m, what is the resultant torque on the wheel?
   A. −18 N•m
   B. −9.0 N•m
   C. 9.0 N•m
   D. 18 N•m

10. A force of 75 N is applied to a lever. This force lifts a load weighing 225 N. What is the mechanical advantage of the lever?
   F. $\frac{1}{3}$
   G. 3
   H. 150
   J. 300

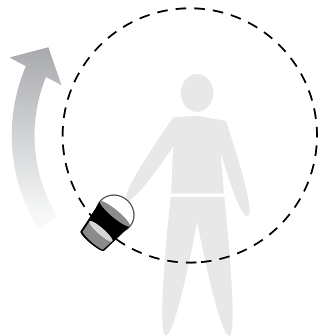
11. A pulley system has an efficiency of 87.5 percent. How much work must you do to lift a desk weighing 1320 N to a height of 1.50 m?
   A. 1510 J
   B. 1730 J
   C. 1980 J
   D. 2260 J

12. Which of the following statements is correct?
   F. Mass and weight both vary with location.
   G. Mass varies with location, but weight does not.
   H. Weight varies with location, but mass does not.
   J. Neither mass nor weight varies with location.

13. Which astronomer discovered that planets travel in elliptical rather than circular orbits?
   A. Johannes Kepler
   B. Nicolaus Copernicus
   C. Tycho Brahe
   D. Claudius Ptolemy

SHORT RESPONSE

14. Explain how it is possible for all the water to remain in a pail that is whirled in a vertical path, as shown below.



15. Explain why approximately two high tides take place every day at a given location on Earth.

16. If you used a machine to increase the output force, what factor would have to be sacrificed? Give an example.

EXTENDED RESPONSE

17. Mars orbits the sun ($m = 1.99 \times 10^{30}$ kg) at a mean distance of 2.28×10^{11} m. Calculate the length of the Martian year in Earth days. Show all of your work. ($G = 6.673 \times 10^{-11}$ N•m^2/kg^2)

Test TIP If you are solving a quantitative problem, start by writing down the relevant equation(s). Solve the equation(s) to find the variable you need for the answer, and then substitute the given data.

CHAPTER 7

Inquiry Lab

Machines and Efficiency

Design Your Own

OBJECTIVES

- **Measure** the work input and work output of several machines.
- **Calculate** the efficiency of each machine.
- **Compare** machines based on their efficiencies, and determine what factors affect efficiency.

MATERIALS LIST

- **balance**
- **C-clamp**
- **cord**
- **dynamics cart**
- **inclined plane**
- **mass hanger**
- **meterstick**
- **pulleys, single and tandem**
- **set of hooked masses**
- **right-angle clamp**
- **support stand**
- **suspension clamp**

In this lab, you will design an experiment to study the efficiency of two types of simple machines: inclined planes and pulleys. In your experiment, you should use each type of machine to use a smaller mass to lift a larger mass. With each setup, you should collect data that will allow you to calculate the work input and the work output of the system. The ratio of the useful work output to the work input is called the *efficiency* of a machine. By calculating efficiency, you will be able to compare the two types of machines.

SAFETY

- **Tie back long hair, secure loose clothing, and remove loose jewelry to prevent their getting caught in moving parts and pulleys. Put on goggles.**
- **Attach string to masses and objects securely. Falling or dropped masses can cause serious injury.**

PROCEDURE

1. Study the materials provided, and design an experiment to meet the goals stated above.

Figure 1

- Choose any angle, but make sure the top of the plane is at least 20 cm above the table.
- Make sure the string is long enough to help prevent the cart from falling off the top of the plane. Attach the mass hanger securely to the end of the string.

2. Write out your lab procedure, including a detailed description of the measurements to take during each step and the number of trials to perform. You may use **Figure 1** as a guide to one possible setup.
3. Ask your teacher to approve your procedure.
4. Follow all steps of your procedure.
5. Clean up your work area. Put equipment away safely so that it is ready to be used again.

ANALYSIS

1. **Organizing Data** For each trial, make the following calculations:
   a. the weight of the mass being raised
   b. the weight of the mass on the string
   c. the work input and the work output
2. **Analyzing Results** In which trial did a machine perform the most work? In which trial did a machine perform the least work?
3. **Organizing Data** Calculate the efficiency for each trial.
4. **Analyzing Results** Is the machine that performed the most work also the most efficient? Is the machine that performed the least work also the least efficient? What is the relationship between work and efficiency?

CONCLUSIONS

5. **Drawing Conclusions** Based on your calculations in item 4, which is more efficient, a pulley system or an inclined plane?
6. **Evaluating Methods** Why is it important to calculate the work input and the work output from measurements made when the object is moving with constant velocity?

EXTENSIONS

7. **Designing Experiments** Design an experiment to measure the efficiency of different lever setups. If there is time and your teacher approves, test your lever setups in the lab. How does the efficiency of a lever compare with the efficiency of the other types of machines you have studied?
8. **Building Models** Compare the trial with the highest efficiency and the trial with the lowest efficiency. Based on their differences, design a more efficient machine than any you built in the lab. If there is time and your teacher approves, test the machine to test whether it is more efficient.